Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Mathematical Methods-II

Subject Code: 5SC04MAM1 **Branch: M.Sc. (Mathematics)**

Semester: 4 **Date : 01/05/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Derive the second form of Euler's equation. **(02)**

b. Find the extremal of $I[y(x)] = \int_1^3 (3x - y)y\,dx$ with $y(1) = 1, y(3) = 9/2$. **(02)**

c. Prove that the shortest distance between two points in a plane is a straight line. **(02)**

d. Show that $y(x) = 2 - x$ is a solution of the integral equation $\int_0^x e^{x-t}\, y(t)\, dt = e^x + x - 1$. **(01)**

Q-2 **Attempt all questions** **(14)**

a. Show that the geodesics on a sphere of radius a are its great circles. (06)

b. Show that the general solution of the Euler's equation for the integral $\int_a^b \frac{1}{y}\sqrt{\left[1 + \left(\frac{dy}{dx}\right)^2\right]}\, dx$ is $(x + h)^2 + y^2 = k^2$, where h and k are constants. (05)

c. Find the extremals of the isoperimetric problem $\int_{x_0}^{x_1} y'^2\, dx$ given that $\int_{x_0}^{x_1} y\, dx = c$, a constant. (03)

OR

Q-2 **Attempt all questions** **(14)**

a. Prove that if the functional $I[y(x)] = \int_{x_1}^{x_2} f(x, y, y')\, dx$ has the extremum value, then the integrand f satisfies the Euler's equation $\frac{\partial f}{\partial x} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. (06)

b. Find the extremal of the functional $I = \int_0^1 (y')^2\, dx$ under the conditions $y(0) = 1, y(1) = 6$ and subject to the constraint $\int_0^1 y\, dx = 3$. (05)

c. Show that the functional $\int_0^1 (2x + x'^2 + y'^2)\ dt,\ x(0) = 1, x(1) = 1.5$ $y(0) = 1, y(1) = 1$ is stationary for $x(t) = \frac{2+t^2}{2}$ and $y(t) = 1$. (03)

Q-3 **Attempt all questions** **(14)**

a. State Leibniz's rule. Prove that (08)

$$\int_a^x \int_a^{x_n} \dots \int_a^{x_2} f(x_1) dx_1 dx_2 \dots dx_n = \frac{1}{(n-1)!} \int_a^x (x-t)^{n-1} f(t)\, dt$$

b. Find the extremals of the functional $\int_0^{\pi/2} (y''^2 - y^2 + x^2)\ dx$ that satisfies the conditions $y(0) = 1, y'(0) = 0, y\left(\frac{\pi}{2}\right) = 0, y'\left(\frac{\pi}{2}\right) = -1$. (06)

OR

Q-3 **Attempt all questions** **(14)**

a. Find the integral equation corresponding to the boundary value problem $y''(x) + \lambda\, y(x) = 0,\quad y(0) = y(1) = 0.$ (08)

b. Solve the functional $I[x(t), y(t)] = \int_0^{\pi/4} (x'y' + 2x^2 + 2y^2)\ dt$ with $x(0) = y(0) = 0$ and $x\left(\frac{\pi}{4}\right) = y\left(\frac{\pi}{4}\right) = 1$. (06)

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a. Convert the integral equation $y(x) = 1 - \int_0^x t\, y(t)\ dt$ into differential equation along with initial condition. (02)

b. State Fredholm integral theorem. (02)

c. Write Hermite's equation and Legendary equation. (02)

d. Define: Separable Kernel. (01)

Q-5 **Attempt all questions** **(14)**

a. Solve the integral equations $\frac{dy}{dx} + 4y + 5\int_0^x y(t)\ dt = e^{-x}, y(0) = 0.$ (07)

b. Obtain the solution of $y(x) = 1 + \lambda \int_0^1 x\, t \cdot y(t)\, dt$ in the form $y(x) = 1 + \frac{3\lambda x}{2(3-\lambda)}$ $(\lambda \neq 3)$. What happens when $\lambda = 3$? (04)

C. Transform the equation $xy'' + (1-x)y' + ny = 0$ to Strum-Liouville equation. (03)

OR

Q-5 **Attempt all questions** **(14)**

a. Solve the integral equation $y(x) = \cos x + \lambda \int_0^{\pi} \sin(x-t)\ y(t)\, dt$. (07)

b. Solve the Abel's integral equation $\int_0^x \frac{y(t)}{(x-t)^{1/2}}\, dt = x + 1.$ (04)

c. Convert the integral equation $y(x) = 1 + \int_0^x (x+t)\, y(t)\, dt$ into differential equation along with initial condition. (03)

Q-6 **Attempt all questions** **(14)**

a. Find the shortest distance between parabola $y^2 = x$ and the straight line $x - 5 = y$. (07)

b. Find the eigenvalues and the corresponding Eigen functions of the differential equation $y'' + \lambda\, y = 0$ on the interval $[0, c]$ with the boundary conditions $y(0) = 0$ and $y(c) = 0$. (07)

OR

Q-6 **Attempt all Questions** **(14)**

a. Find the extremum distances of the interior point $(1, 0)$ from the curve $4x^2 + 9y^2 = 36$. (07)

b. Find the eigenvalues and the corresponding Eigen functions of the differential equation (07)

$$4(e^{-x}y')' + (1+\lambda)e^{-x}y = 0, y(0) = 0, y(1) = 0$$